Veer Narmad South Gujarat University, Surat.

M. Sc. Part II
(Mathematics)

w. e. f. 2002

Paper 501 – Advanced Functional Analysis

Fundamental theorem for Normed & Banach spaces:

Zorn's Lemma, Hahn-Banach theorem, Generalised Hahn-Banach theorem, Hahn-Banach theorem for Normed spaces, Application to bounded linear functionals on C[a, b], Riesz's theorem for functionals on C[a, b], Adjoint operator and its norm, Relation between adjoint and Hilbert adjoint operators, Reflexive spaces, Baire's Category theorem and Uniform Boundedness theorem with applications, Strong and Weak convergence, Convergence of sequences of operators and functionals, Open Mapping theorem, Closed linear operators and Closed Graph theorem.

Spectral Theory of Linear Operators in Normed Spaces:

Spectral theory in finite dimensional normed spaces, Eigen values, Eigen vectors, Eigen spaces, Spectrum and resolvant set of matrix-definitions, Eigen values of an operator, Definition of Regular value, Point, Continuous and Residual spectrum, Spectral properties of bounded linear operators, Properties of Resolvant and Spectrum, Holomorphy and Local holomorphy, Use of complex analysis in spectral theory.

Compact Linear Operators on Normed Spaces and their Spectrum:

Compact linear operators on normed spaces and their properties, Spectral properties of compact linear operators on normed spaces.

Spectral Theory of Bounded Self Adjoint Linear Operators:

Spectral properties of bounded self-adjoint linear operator, Theorem on eigenvalues and eigenvectors, Theorems on resolvent set and spectrum, Residual spectrum theorem, Positive operators, Product of positive operators, Projection operators and their properties.

Unbounded Linear Operators on Hilbert Space:

Unbounded linear operators and their Hilbert adjoint operators, Symmetric and self-adjoint linear operators, Closed linear operators Closure of Hilbert adjoint, Spectral properties of Hilbert adjoint, Spectral representation of unitary operators and self-adjoint operators, Multiplication and Differential operators, Riesz's theorem, Sequilinear form and Riesz's representation theorem, Hilbert adjoint operator and its properties, Self-adjoint, Unitary and Normal operators.

References:

1. Introductory Functional Analysis with applications by Erwin Kreyszig (2ed) John Wiley and Sons., 1978.
2. Functional Analysis by B. V. Limaye. (2ed). New-Age Int. Pvt. Ltd.
3. Introduction to Topology and Modern Analysis by G. F. Simmons. Mc Graw Hill Book Co.
4. Functional Analysis by Koftman and Patric.
5. Functional Analysis by Sudarshan Nanda. Wiley Eastern.
6. Functional Analysis by A. H. Siddiqui. Prentice Hall of India Ltd.
7. Functional Analysis by Yosida K. (3rd ed). Springer-Verlag.
8. Normed Linear Spaces by Day M. M. 1973. (3rd ed). Springer-Verlag.

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M. Sc. Part II
(Mathematics)

Paper 502 – Advanced Abstract Algebra

Groups-Normal and Subnormal series, Composition Series. Jordan-Holder theorem. Solvable groups. Nilpotent groups.

Canonical Forms-Similarity of linear transformations. Invariant subspaces. Reduction to triangular forms. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation. The primary decomposition theorem. Jordan blocks and Jordan forms.

Cyclic modules. Simple modules. Semi-simple modules. Schuler's Lemma. Free modules.

Field theory-Extension fields. Algebraic and transcendental extensions. Separable and inseparable extensions. Normal extensions. Perfect fields. Finite fields. Primitive elements. Algebraically closed fields. Automorphisms of extensions. Galois extensions. Fundamental theorem of Galois theory. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

Noetherian and artinian modules and rings-Hilbert basis theorem. Wedderburn-Artin theorem. Uniform modules, primary modules, and Noether-Lasker theorem.

Smith normal form over a principal ideal domain and rank.

Fundamental structure theorem for finitely generated modules over a principal ideal domain and its applications to finitely generated abelian groups. Rational canonical form. Generalised Jordan form over any field.

References:

1. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (2nd Edition), Cambridge University Press, Indian Edition, 1997.
2. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
3. M. Artin, Algebra, Prentice-Hall of India, 1991.
4. P. M. Cohn, Algebra, Vols. I, II & III, John Wiley & Sons, 1982, 1989, 1991.
5. N. Jacobson, Basic Algebra, Vols. I & II, W. H. Freeman, 1980 (also published by Hindustan Publishing Company).
6. S. Lang, Algebra, 3rd Edition, Addison-Wesley, 1993.
7. I. S. Luther and I. B. S. Passi, Algebra, Vol. I-Groups, Vol. II-Rings, Narosa Publishing House (Vol. I-1996, Vol. II-1999).
8. D. S. Malik, J. N. Mordeson, and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw-Hill, International Edition, 1997.
9. K. B. Datta, Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
10. S. K. Jain, A. Gunawardena and P. B. Bhattacharya, Basic Linear Algebra with MATLAB, Key College Publishing (Springer-Verlag), 2001.
11. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice-Hall of India, 2000.
12. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
13. I. Stewart, Galois Theory, 2nd Edition, Chapman and Hall, 1989.
14. J. P. Escofier, Galois Theory, GTM Vol. 204, Springer, 2001.
15. T. Y. Lam, Lectures on Modules and Rings, GTM Vol. 189, Springer-Verlag, 1999.
16. D. S. Passman, A Course in Ring Theory, ‘Wadsworth and Brooks/Cole Advanced Books and Softwares, Pacific Groves, California, 1991.

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M. Sc. Part II
(Mathematics)

Paper 503 – Advanced Linear Algebra

Linear Operators:

Definition, Null space and Range, Rank-Nullity theorem, Operator inverse, Applicaion to matrix theory, Computation of range and null spaces of a matrix, Matrix of an operator, Operator algebra, Change of basis, Similar matrices and applications.

Inner Product Spaces:

Definitions and examples, Norms, Orthogonal sets, Fourier coefficients and Parseval's identity,Gram-Schmidt process and QR factorization, Approximation and orthogonal projection, Applications of projection theory and orthogonal complements.

Diagonalizable Linear Operators:

Definition of Eigenvalues and Eigenvectors, Spectrum and eigen spaces of an operator, Properties of characteristic polynomial, Geometric and algebraic multiplicities, Linear operator with an eigen basis, Functions of diagonalizable operators, First order matrix differential equations, Estimate of eigen value and application to finite difference equations.

Normal Operators:

Adjoints and classification of operators, The spectral theorem, Application to matrix theory, Extremum principles for Hermitian operators,The power method for dominant eigen values and eigen vectors with secondary approximation, Inverse power method and subspace methods.

References:

1. Linear Algebra with Applications by J. T. Scheick. McGraw Hill, International Edition, 1997.
2. Matrix Algebra by S. Biswas. New age Int. Pub. 2nd ed. 1997.
3. Linear Algebra by A. R. Rao & P. Bhima Shankaram. Tata McGraw Hill Pub. Co. Ltd. New Delhi. 1996.
4. Principles and Techniques of Applied Mathematics by B. Friendman. Dover, 1990, NY.
5. Theory of Matrices with Applications by P. Lancaster & M. Tismenetsky. Academic Press, 1985, 2nd ed. NY.

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M. Sc. Part II
(Mathematics)

Paper 5005 – Integral Transforms

OPTIONAL PAPERS

Fourier Transforms:

Introduction, Basic concepts and definitions, The Fourier Integral formulae, Definition and examples of Fourier transforms, Basic properties of Fourier transforms, Applications of Fourier transforms to ordinary differential equations; integral equations and partial differential equations, Fourier cosine and sine transforms with examples, Properties of Fourier cosine and sine transforms, Application of Fourier cosine and sine transforms to partial differential equations and evaluation of definite integrals.

Laplace Transforms:

Introduction and definition of Laplace transforms with examples, Existance condition and basic properties of Laplace transforms, The convolution theorem and properties of convolution, Differentiation and integration of Laplace transforms, The inverse Laplace transforms and examples, Tauberian theorem and Watson’s lemma, Laplace transforms of fractional integrals and fractional derivatives, Application of Laplace transforms to ordinary and partial differential equations; initial and boundary value problems and Integral equations; Evaluation of definite integral and solution of difference as well as differential equations.

Finite Laplace Transforms:

Introduction, Definition of finite Laplace transforms with examples, Basic operational properties of finite Laplace transforms, Application of finite Laplace transforms and Tauberian theorem.

Finite Fourier Cosine and Sine Transforms:

Introduction and definition of finite cosine and sine transforms with examples, Basic properties of finite Fourier cosine and sine transforms, Application of finite Fourier cosine and sine transforms.

References:

1. Integral Transforms and their Applications by Lokenath-Debnath. CRC Publications, 1995.
2. The Use of Integral Transforms by Inn Sneddon. Tata McGraw Hill Publication, 1979.
3. Integral Transforms and their Applications by B. Davies. Springer-Verlag, AMS Vol. 25, 1978.
4. Complex Analysis and Laplace Transforms by Le Paze, TMH Pub.
5. Mathematical Methods in Physical Sciences by M. L. Boas. John Wiley & Sons, 1983.
6. Integral Transforms by P. K. Gupta. Krishna Prakashan Mandir, Meerut, 1990.

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(Mathematics)

Paper 5006 – Advanced Integral Transforms

<u>Hankel Transforms</u>:

Introduction and definition of Hankel transforms with examples, Operational properties of the Hankel transforms and its application to partial differential equations.

<u>Finite Hankel Transforms</u>:

Introduction and definition of the finite Hankel transforms with examples, Basic operational properties and applications of finite Hankel transforms.

<u>Mellin Transforms</u>:

Introduction and definition of Mellin transforms with examples,Basic operational properties and applications of the Mellin transforms, Mellin transforms of the Weyl fractional integrals and weyl fractional derivatives, Application of Mellin transforms to summation of series.

<u>Z-Transforms</u>:

Introduction, Dynamic linear systems and Impulse response, Definition of the Z-transforms and examples, Basic operational properties, The inverse Z-transform and examples, Application of Z-transforms to finite difference equations.

<u>Hilbert and Stieltjes Transforms (HST)</u>:

Introduction and definition of HST with examples, Basic operational properties of HST, Hilbert transform in the complex plane and its applications, Inverse theorem for Stieltjes transform and its application, Asymptotic expansion of of the one sided Hilberet transform, The generalised Stieltjes transform, Basic properties of the generalised Stieltjes transforms with applications.

References:

1. Integral Transforms and their Applications by Lokenath-Debnath. CRC Publication, 1995.
2. The Use of integral Transforms by Inn Sneddon. Tata McGraw Hill, 1979.
3. Integral Transforms and their Applications by B. Davies.Springer-Verlag, Vol. 25, 1978.
4. Mathematical methods in Physical Sciences by Boas M. L.John Wiley & Sons, 1983.
5. Integral Transforms by Gupta P. K. Krishna Prakashan, Meerut, 1990.

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M. Sc. Part II
(Mathematics)

Paper 5007 – Special Functions

Infinite Products:

Introduction and definition, Necessary and sufficient condition for convergence, absolute convergence and uniform convergence.

The Hyper Geometric Function:

The function F(a,b; c;z), A simple integral formula, F(a,b;c;1) as a function of parameters, Evaluation of F(a,b;c;1), The continuous function relation, The Hypergeometric differential equations and their logarithmic solutions, Elementary series manipulations, Simple transformations, Relation between function of z and that of (1 – z), A quadratic transformation, Kummar's theorem, Some additional properties.

Bessel's Functions:

Definition of $J_n(z)$, Bessel's differential equation, Differential and pure recurrence relation, Generating functions, Bessel's integrals index half an odd integer, Modified Bessel's functions, Neumann polynimial and Neumann series.

Legendre Polynomials:

The generating function, Differential and pure recurrence relations, Legendre's differential equation, The Rodrigues formula, Batemann generating function, Additional generating functions, Hypergeometric forms of $P_n(x)$, Brafman's generating function, Properties of $P_n(x)$ with more generating functions,Laplace first integral form, Bounds on $P_n(x)$, Orthogonality theorem, Expausition theorem, expansion of x^n and expansion of analytic functions.

Hermite Polynomials:

Definition of $H_n(x)$, Recurrence relations, Rodrigues formula and generating functions, integrals, Hermite polynomial as a {F_o, orthogonality, Expansion of polynomials and more about generating functions.

Leguerre Polynomials:

A Polynomial $L_n^{(\alpha)}(x)$, Generating functions and recurrence relations, The differential equation, Orthogonality, Expausition of polynomial and special properties, Other generating functions, The simple Leguerre polynomial.

References:

1. Special Functions by Rainville E.D. McMillan, New York, 1960.
2. Special functions of Mathematical Physics and Chemistry by Sneddon I. N.Oliver Boyd, 1961.
3. A Treatise on the theory of Bossel's functions by Watson G. N.Cambridge University Press, 1931.
4. Special Functions and their Applications by Ledebev N. N. Dover Pub. 1972.
5. Special Functions by Saxena R. K. and Gokhroo D. C. Khanna Pub.

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Paper 5008 – Advanced Special Functions

The Hyper Geometric Function:

The function F(a,b; c;z), A simple integral formula, F(a,b;c;1) as a function of parameters, Evaluation of F(a,b;c;1), The continuous function relation, The Hypergeometric differential equations and their logarithmic solutions, Elementary series manipulations, Simple transformations, Relation between function of z and that of (1 – z), A quadratic transformation, Kummar's theorem, Some additional properties.

Generalised Hypergeometric functions:

The function ${}_pF_q$. The Exponential and Binomial functions, Differential equation and its various solutions, The continuous function relations with simple integral, ${}_pF_q$ with unit argument, Saalschutz' theorem, Whipple's theorem, Dixon's theorem, Contour integrals of Barnes' type, The Barnes integrals and the function ${}_pF_q$ with some useful integral.

The Confluent Hypergeometric Function:

Barie properties of ${}_1F_1$, Kummar's first and second formula.

Generating Functions:

Concept of the generality function, The generating function of the form $G(2xt – t^2)$, Sets generated by $e^t\psi(xt)$, The generating function $A(t)\exp[^{-xt}/_{(1-t)}]$, Another class of generating functions and its extension.

Orthogonal Polynomials:

Simple sets of polynomials, Orthogonality and equivalent condition for orthogonality, Zeros of orthogonal polynomials, Expansion of polynomials, The three – term recurrence formula, The Cristoffel – Darbaux formula, Normalization, Bessel's inequality.

Jacobi Polynomials:

The Jacobi polynomials, Bateman's generating function, The Rodrogues formula and orthogonality, Differential and pure recurrence relations, Mixed relations, Appell's functions of two variables, Elementary generating functions, Brafman's generating functions, Expansion in series of polynomials.

References:

1. Special Functions by Rainville E.D. McMillan, New York, 1960.
2. Special functions of Mathematical Physics and Chemistry by Sneddon I. N.Oliver Boyd, 1961.
3. A Treatise on the theory of Bossel's functions by Watson G. N. Cambridge University Press, 1931.
4. Special Functions and their Applications by Ledebev N. N. Dover Pub. 1972.
5. Special Functions by Saxena R. K. and Gokhroo D. C. Khanna Pub.

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M. Sc. Part II
(Mathematics)

Paper 5021 – Computational Fluid Dynamics

- Basic Ideas of Computational Fluid Mechanics
- Governing Equations of Fluid Mechanics
- Classification of Quasi-Linear PDEs
- Additional Issues of CFD: Space-Time Resolution of flows
- Discretization of PDEs
- Solution methods for Parabolic PDEs and their Analysis
- Solution methods for Elliptic PDEs
- Solution of Hyperbolic PDEs
- Grid Generation
- Finite Difference Methods for PDE & Finite Volume Methods
- Solution of Navier-Stokes Equation
- ODE and PDE Solver (Practicals)

References:

1. Anderson: Computational Fluid Dynamics: The Basics with Applications, McGraw Hill.
2. T. K. Sengupta: Fundamentals of Computational Fluid Dynamics, University Press.
3. P. Wesseling: Principles of Computational Fluid Dynamics.
4. Maurice Holt: Numerical Methods in Fluid Dynamics.
5. Bosk T. K.: Computational Fluid Dynamics.
6. Reddy J. N.: An Introduction to the FFM.
7. Baker A. J.: Finite Element Methods for Computational Fluid Dynamics.
8. Batchelor G. K.: An Introduction to Fluid Dynamics, Cambridge University Press.
9. Chorin: Mathematical Introduction to Fluid Mechanics, Springer Verlag.
10. Versteeg, H. K. and Malalasekera, W.: An Introduction to Computational Fluid Dynamics: The Finite Volume Method, Addison-Wesley, 1996.
11. Hoffmann, K. A. Dietiker, J. F., and Devahastin, A.: Student Guide to CFD, Volume I, Engineering Education Systems, 2001.

12. Hoffmann, K. A. Dietiker, J. F., and Chiang, S. T.: Student Guide to CFD, Volume II, Engineering Education Systems, 2001.
13. Ferziger, J. H. and Peric, M.: Computational Methods for Fluid Dynamics, 3rd ed., Springer, 2001.
14. Fletcher, C. A. J.: Computational Techniques for Fluid Dynamics, Vol. I & II, Springer-Verlag, 1988.
15. Fletcher, C. A. J. and Srinivas, K.: Computational Techniques for Fluid Dynamics: A Solutions Manual, Springer-Verlag, 1992.
16. Abbot, M. B. and Basco, D. R.: Computational Fluid Dynamics: An Introduction for Engineers, John Wiley & Sons, 1989.
17. Lomax, H., Pulliam, T. H., and Zingg, D. W.: Fundamentals of Computational Fluid Dynamics, Springer-Verlag, 2001.
18. Patankar, S. V.: Numerical Heat Transfer and Fluid Flow, Hemisphere Pub. Corporation, 1980.
19. Shaw, C. T.: Using Computational Fluid Dynamics, Prentice Hall, 1992.

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M. Sc. Part II
(Mathematics)

Paper 5022 – Numerical Analysis & Mathematical Software

Understanding MATLAB

- Introduction to MATLAB, Matlab Windows, Symbolic Calculations, Basic Features
- Files and Directory management, File I/O operations
- Working with Arrays of Numbers
- Matrices and Vectors, Operations on Matrices
- Arithmetic, Relational, Logical, operations, Elementary math functions
- Script and Functions, Subfunctions
- Applications in Linear Algebra, Curve Fitting and Interpolation, Numerical Integration, Ordinary Differential Equations
- 2-D, 3-D Graphics

Numerical Analysis

- Solution of algebraic and transcendental equations
- Numerical Solution methods for Differential Equations
- Solution of System of Linear Equations: Matrix inversion, Jordan's method
- LU and Cholesky factorisations
- Pivoting, Gauss Elimination method, Jacobi's, Gauss-Seidel method
- Algebraic Eigen Value Problem, Properties of eigen values, eigen vectors, Power method, inverse power method, Given's method, Schur and Gershgorins theorem
- Least square polynomial approximation
- Numerical Solution of ODE: Runge Kutta methods, Milne Simpsons' method
- Finite Difference Methods for ODE
- System of non-linear equations: Newton Raphson's method.

References:

1. S. Balachandra Rao, C. K. Shantha, Numerical Methods with Programs in BASIC, FORTRAN, Pascal and C++, University Press.
2. Rudra Pratap, Getting Started with MATLAB – A Quick Introduction for Scientists and Engineers, Oxford University Press, 2004.
3. Duane Hanselman and Bruce Littlefied, Mastering Matlab, A Comprehensive tutorial and reference.
4. Delores M. Etter, Engineering Problem Solving with Matlab, Prentice Hall, 1993.
5. Gustafsson and Bergman, Matlab for Engineers Explained, Springer, 2003.

6. C. E. Froberg: Introduction to Numerical Analysis, Addison Wesley Publishing Company, Sixth Ed., 1981.
7. S. S. Sastri: Introductory Methods of Numerical Analysis, Prentice Hall of India, New Delhi, 1997.
8. Conte S. D. and Carl deboor: Elementary Numerical Analysis: an algorithmic approach, Mc Graw Hill company, Third Ed., 1981.
9. M. K. Jain: Numerical Analysis for Scientists and Engineers, New Age International Ltd. Pub., 1992.
10. E. Hairer, E. P. Norsett and G. Wanner: Solving ordinary differential equations I and II, Springer Series in Computational Mathematics, 8, Springer, Berlin, 1993.